

Gediminas Juzeliūnas and Marius Mašalas  
*Institute of Theoretical Physics and Astronomy, A. Goštauto 12,*  
*Vilnius 2600, Lithuania*  
 (December 5, 2000)

We study the absorption spectrum for a strongly degenerate Fermi gas confined in a harmonic trap. The spectrum is calculated using both the exact summation and also the Thomas-Fermi (TF) approximation. In the latter case, relatively simple analytical expressions are obtained for the absorption lineshape at large number of trapped atoms. At zero temperature, the approximated lineshape is characterized by a  $(1 - z^2)^{5/2}$  dependence which agrees well with the exact numerical calculations. At non-zero temperature, the spectrum becomes broader, although remains non-Gaussian as long as the fermion gas is degenerate. The changes in the trap frequency for an electronically excited atom can introduce an additional line broadening.

32.70.Jz, 42.50.Fx, 32.80.-t

In recent years there has been a great deal of interest in the dilute gas of trapped atoms cooled to temperatures below  $1 \mu\text{K}$ . At such low temperatures, an important role is played by the quantum statistics of atoms. Bosons tend to occupy the lowest translational level of the trap to form the Bose - Einstein condensate [1-3]. On the other hand, fermions can still occupy many trap levels (predominantly up to the Fermi level), as it was observed recently [4]. The physical properties of a Fermi gas (such as the specific heat) depend on the number of atoms in the system [5,6], as well as on the trap anisotropy [6].

The optical spectroscopy has proven to be useful in getting information about cold atomic gases [1-3,7-10]. The Fermi-Dirac (FD) statistics is known to change the optical response of the system compared to the classical one. The signatures of quantum degeneracy emerge in the scattering spectra of homogeneous [10] and trapped [7] Fermi gases. Effects of quantum statistics are also featured in the scattering of short laser pulses from a trapped Fermi gas [9]. Furthermore, the spontaneous emission appears to be inhibited in a cold Fermi gas [11].

The effects of quantum degeneracy should manifest in the absorption spectra as well. The aim of the present paper is to investigate absorption by a cold Fermi gas confined in a harmonic trap. The analysis concentrates on the degenerate Fermi gas (i.e. very low temperatures), for which the quantum statistics of the atoms plays an important role. The theory involves exact calculations, as well as the Thomas - Fermi (TF) approximation. Consequently, the absorption spectrum is analyzed both for small and large numbers of trapped atoms.

Consider a gas of Fermi atoms confined in a harmonic

trap. The harmonic approximation is relevant for the traps used in recent experiments [4]. We shall neglect atomic collisions, since the s-wave collisions are forbidden between the spin - polarized fermions. Consequently, one can make use of the following one - atom Hamiltonian:

$$H_{1-at} = |g\rangle H_g |g\rangle + \sum_{ex} |ex\rangle (\hbar\omega_{0,ex} + H_{ex}) \langle ex|, \quad (1)$$

where  $|g\rangle$  and  $|ex\rangle$  represent the ground and an excited electronic state of an atom,  $\hbar\omega_{0,ex}$  is the excitation energy. It is noteworthy that the ground electronic level of the fermion atom has a number of magnetic sublevels over which the summation is to be carried out in the Hamiltonian (1). However, such a summation is not necessary if the atoms are spin - polarized, as it is the case in the experiment by DeMarco and Jin on  $^{40}\text{K}$  atoms [4]. Here also  $H_g$  ( $H_{ex}$ ) is the Hamiltonian for the translational motion of a trapped atom in the ground (excited) electronic state:

$$H_{g,ex}(\mathbf{r}, \mathbf{p}) = \frac{p^2}{2M} + \frac{M\Omega_{g,ex}^2 (x^2 + \lambda_y^2 y^2 + \lambda_z^2 z^2)}{2}, \quad (2)$$

where  $\mathbf{p} = -i\hbar\nabla$  is the momentum operator,  $M$  is the atomic mass,  $\Omega_g$  ( $\Omega_{ex}$ ) is the frequency of the translational motion along the x axis if the atom is in the ground (excited) electronic state, and the dimensionless parameters  $\lambda_y$  and  $\lambda_z$  describe the extent of anisotropy of the trap. Note that the frequency  $\Omega_g$  can be generally different from  $\Omega_{ex}$  due to the changes in the magnetic moment of the atom following its transition to an excited electronic state. The effects related to this fact will be explored using the TF approximation.

The lineshape of the absorption spectrum is given by

$$I(\omega) = \sum_{i,f} \rho_i |\langle f|V|i\rangle|^2 \delta(\omega - \omega_{fi}). \quad (3)$$

Here  $|i\rangle \equiv |g\rangle |\mathbf{n}\rangle_g$  and  $|f\rangle \equiv |ex\rangle |\mathbf{m}\rangle_{ex}$  are the initial and final states of an atom,  $|\mathbf{n}\rangle_g \equiv |n_x, n_y, n_z\rangle_g$  and  $|\mathbf{m}\rangle_{ex} \equiv |m_x, m_y, m_z\rangle_{ex}$  are the atomic translational states characterized by the energies

$$\varepsilon_{\mathbf{n}}^{g,ex} = \hbar\Omega_{g,ex} (n_x + \lambda_y n_y + \lambda_z n_z) + \varepsilon_{\mathbf{0}}^{g,ex}, \quad (4)$$

where  $\varepsilon_{\mathbf{0}}^{g,ex} = \hbar\Omega_{g,ex} (1 + \lambda_y + \lambda_z)/2$ , and  $\omega_{fi} = \omega_{0,ex} + (\varepsilon_{\mathbf{m}}^{ex} - \varepsilon_{\mathbf{n}}^g)/\hbar$  is the transition frequency. Here also  $\rho_i \equiv \rho(\varepsilon_{\mathbf{n}}^g) = [\exp(\beta\varepsilon_{\mathbf{n}}^g - \beta\mu) + 1]^{-1}$  is the FD distribution function for the trapped atoms,  $\mu$  is the chemical potential and  $\beta = 1/k_B T$ . The operator

$$V = \sum_{ex} \{ |ex\rangle d_{ex} \exp(i\kappa x) \langle g| + h.c. \} \quad (5)$$

describes the interaction between an atom and the electromagnetic field propagating along the  $x$  axis,  $\kappa$  is the wave number of the light, and  $d_{ex}$  is the atomic transition dipole moment along the polarization of the light.

Consider first the absorption spectrum using the exact summation over the translational levels. At this stage, it is assumed that  $\Omega_g = \Omega_{ex} = \Omega$ , yet the trap can still be anisotropic. At zero temperature, only the levels with  $n_x + \lambda_y n_y + \lambda_z n_z \leq n_F = E_F/\hbar\Omega$  are occupied by the atoms, where  $E_F \equiv \mu|_{T=0}$  is the Fermi energy. In such a situation, the absorption lineshape takes the form:

$$I(\omega) = \sum_{ex} |d_{ex}|^2 \sum_{n_x=0}^{n_F} \sum_{m=-n_x}^{\infty} K_x \delta(\omega - \omega_{0,ex} - m\Omega) \times n_x! (n_x + m)! e^{-\alpha^2} (\alpha^2)^{2n_x+m} \times \left( \sum_{j=0}^{j_x} \frac{(-\alpha^{-2})^j}{j! (n_x - j)! (n_x + m - j)!} \right)^2, \quad (6)$$

with  $j_x = \min(n_x, n_x + m)$ , where  $\alpha = \kappa(\hbar/2M\Omega)^{1/2}$ , and the factor  $K_x = \sum_{n_y=0}^{\lfloor \frac{1}{\lambda_y}(n_F - n_x) \rfloor} \sum_{n_z=0}^{\lfloor \frac{1}{\lambda_z}(n_F - n_x - \lambda_y n_y) \rfloor} 1$  represents a number of occupied translational states  $|n_x, n_y, n_z\rangle$  for a fixed value of  $n_x$ , the brackets [...] labeling the integer part of a number. The Fermi number  $n_F$  is determined by the condition  $\sum_{n_x=0}^{n_F} K_x = N$ , where  $N$  is the number of trapped atoms. The factor  $K_x$  reflects the trap geometry. For traps with a cylindric symmetry ( $\lambda_y = \lambda_z = \lambda$ ), one finds  $K_x = ([q_x] + 1)([q_x] + 2)/2$ , where  $q_x = (n_F - n_x)/\lambda$ . For an anisotropic trap of a cigar shape ( $\lambda_y, \lambda_z \gg 1$ ), one has  $K_x = 1$  provided the number of trapped atoms is small enough ( $n_F < \lambda_y, \lambda_z$ ). In such a situation, the trap becomes one-dimensional (1D), giving  $n_F = N - 1$ . On the other hand, for an isotropic three-dimensional (3D) trap ( $\lambda_y = \lambda_z = 1$ ), one arrives at  $K_x = (n_F - n_x + 1)(n_F - n_x + 2)/2$ . If the number of atoms is sufficiently large ( $n_F > \lambda_y, \lambda_z$ ), the anisotropic traps of cigar shape ( $\lambda_y, \lambda_z \gg 1$ ) are no longer one dimensional, since the Fermi energy is then greater than the energy of the translational quanta in the  $y$  and  $z$  directions. Such a situation corresponds to the recent experiment by DeMarco and Jin [4].

Figure 1 shows the absorption lineshapes for various degrees of the trap anisotropy in the case where  $\lambda_y = \lambda_z = \lambda$ . A single excited electronic state  $|ex\rangle$  has been taken into account in these and the subsequent figures. Oscillations are seen clearly in the thick solid line representing a purely 1D case ( $\lambda = 20$ ), as well as in the thin one corresponding to an anisotropic 3-D case ( $\lambda = 5$ ). This can be related to the oscillations of the density of Fermi atoms in the one-dimensional [12] and anisotropic three dimensional traps [6] at a sufficiently small number of the trapped atoms. Oscillations do not appear in the lineshape of an isotropic 3D trap ( $\lambda = 1$ ).

Note that in contrast to a single trapped atom [13], the zero-temperature lineshape of the trapped Fermi gas has a cut off at the frequencies smaller than  $\omega_{0,ex}$ . This can be explained by the fact that fermions occupy excited translational levels of the trap at  $T=0$  (up to the Fermi level), so that optical absorption can be accompanied by a decrease in the translational energy of the atoms.

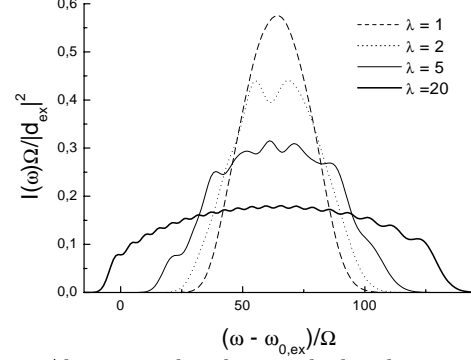


FIG. 1. Absorption lineshape calculated using Eq.(6) for  $N = 20$ ,  $T = 0$  and  $\alpha = 8$ . The singular contributions due to the delta functions have been smoothened.

When  $N$  or  $T$  is increasing, the behavior of a quantum system becomes more similar to that of the classical one. To get analytical formulas for the absorption lineshape at arbitrarily large values of  $N$  and  $T$ , we shall make use of the semiclassical Thomas-Fermi (TF) approximation. In the TF approximation, the state of an atom is labeled by the radius-vector  $\mathbf{r}$  and wave vector  $\mathbf{k} = \mathbf{p}/\hbar$  (see e.g. refs. [5,6]). The density of such states in the six-dimensional phase space  $(\mathbf{r}, \mathbf{k})$  is  $(2\pi)^{-3}$ . The number density of the fermion atoms in the phase space is:

$$\rho(\mathbf{r}, \mathbf{k}, T) = \frac{1}{(2\pi)^3} \frac{1}{\exp\{\beta H_g(\mathbf{r}, \hbar\mathbf{k}) - \beta\mu\} + 1}, \quad (7)$$

where the chemical potential  $\mu$  is related to the number of trapped atoms via the normalization condition  $\int d^3\mathbf{r} d^3\mathbf{k} \rho(\mathbf{r}, \mathbf{k}, T) = N$ .

Applying the TF approximation, the lineshape reads:

$$I(\omega) = \sum_{ex} |d_{ex}|^2 \int d^3\mathbf{r} d^3\mathbf{k} \rho(\mathbf{r}, \mathbf{k}, T) \delta(\omega - \omega_{\mathbf{r},\mathbf{k}}), \quad (8)$$

where  $\omega_{\mathbf{r},\mathbf{k}} = M(\Omega_{ex}^2 - \Omega_g^2)(x^2 + \lambda_y^2 y^2 + \lambda_z^2 z^2)/2\hbar + \hbar k_x \kappa/M + \hbar \kappa^2/M + \omega_{0,ex}$  is the transition frequency. If  $\Omega_g = \Omega_{ex}$ , the frequency  $\omega_{\mathbf{r},\mathbf{k}}$  does not depend on the atomic position  $\mathbf{r}$ , so the lineshape is determined exclusively by the momentum distribution function:

$$n(\mathbf{k}, T) = \int d^3\mathbf{r} \rho(\mathbf{r}, \mathbf{k}, T). \quad (9)$$

For  $T = 0$  the distribution function is given by [5]:

$$n(\mathbf{k}, 0) = \frac{8N}{\pi^2 K_F^3} \left(1 - \frac{k^2}{K_F^2}\right)^{3/2}, \quad (10)$$

where  $K_F = (2ME_F/\hbar^2)^{1/2}$  is the maximum momentum of the trapped Fermi atoms at zero temperature, and  $E_F = \hbar\Omega_g (6\lambda_y\lambda_z N)^{1/3}$  is the Fermi energy. Note that the momentum distribution is isotropic even though the trap is anisotropic [5]. This leads to an isotropic absorption lineshape in the case where  $\Omega_{ex} = \Omega_g$ . Applying the distribution function (10), one arrives at the  $(1 - z^2)^{5/2}$  behavior of the lineshape if  $\Omega_{ex} = \Omega_g = \Omega$ :

$$I(\omega) = \frac{16N}{5\pi\Delta} \sum_{ex} |d_{ex}|^2 \left[1 - (\omega - \omega_{max})^2 / \Delta^2\right]^{5/2}, \quad (11)$$

where the central frequency  $\omega_{max} = \omega_{0,ex} + \alpha^2\Omega$  is shifted by the recoil frequency  $\omega_{rec} = \alpha^2\Omega$  as compared to  $\omega_{0,ex}$ ,

$$\Delta = \alpha\Omega (6N\lambda_y\lambda_z)^{1/6} \quad (12)$$

being the spectral half-width. In the experiment by De-Marco and Jin [4] using trapped  $^{40}\text{K}$  atoms,  $\Omega = 2\pi \times 19$  Hz,  $N = 7 \times 10^5$  and  $\lambda \approx 7$ , giving  $\alpha \approx 36$  for  $\omega = 4 \times 10^{16}$  Hz. Consequently one has  $\Delta \approx 6 \times 10^5$  Hz. This is less than the typical radiative linewidths for free atoms. Yet, for trapped fermions the spontaneous emission is suppressed [11], so the Doppler broadening can be dominant.

The approximated lineshape (11) depends on the trap anisotropy exclusively through the characteristic frequency  $\Omega_{char} = \Omega (\lambda_y\lambda_z)^{1/3}$  which is a measure of the trap hardness. The bigger  $\Omega_{char}$  is, the tighter is the trap, and the broader is the absorption spectrum. In fact, the maximum momentum of the atoms is larger in tighter traps (for the same number of trapped atoms) leading to the increase in the Doppler broadening. For instance, compared to an isotropic trap ( $\lambda_y = \lambda_z = 1$ ), the spectrum of a squeezed trap ( $\lambda_y, \lambda_z > 1$ ) is broader.

Exact and approximated lineshapes are presented in Figs. 2 and 3. For an isotropic trap (Fig. 2), the agreement appears to be very good, even though the number of atoms  $N$  is rather small. Deviations are seen only in the tails of the spectrum corresponding to the periphery of the fermion cloud. In such an area, the fermion density becomes small and the TF approximation fails [5]. For anisotropic traps, the exact spectrum undergoes some oscillations about the approximated one even for relatively large values of  $N$  (see Fig. 3). In fact, the energy of translational quanta depends now on the specific directions of atomic motion, so a larger number of trapped atoms is needed to populate substantially the translational levels in all three directions.

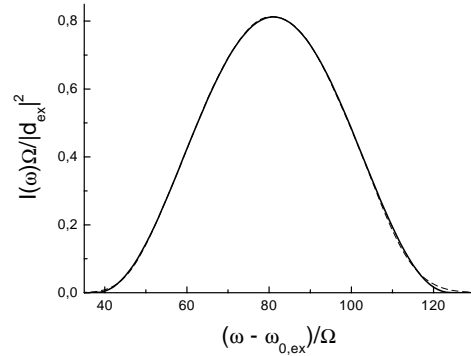


FIG. 2. Absorption lineshape at  $T = 0$  calculated using the exact summation (dashed line) and the TF approximation (solid line) for  $\alpha = 9$ ,  $N = 35$  and  $\lambda_x = \lambda_y = \lambda = 1$ .

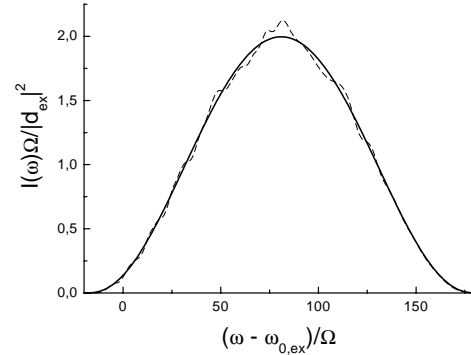


FIG. 3. The same as in Fig. 2, but for  $N = 196$  and  $\lambda = 5$ .

Consider next the situation where  $T \neq 0$  and the frequencies  $\Omega_g$  and  $\Omega_{ex}$  are not necessarily equal. The lineshape (8) takes then the form for an isotropic trap:

$$I(\omega, T) = \sum_{ex} \frac{|d_{ex}|^2}{16\pi\alpha^6\Omega^6 p^{5/2}} \int_0^\infty y^2 dy \ln \left\{ 1 + \exp [\beta\mu - y^2 - p(\omega - \omega_{max} + m_{ex}y^2/\beta\hbar)^2] \right\}, \quad (13)$$

where  $m_{ex} = 1 - \Omega_{ex}^2/\Omega_g^2$ ,  $p = \beta\hbar/4\alpha^2\Omega$  and  $\Omega \equiv \Omega_g$ . If  $\Omega_g = \Omega_{ex}$ , the result (13) can be extended readily to anisotropic traps. In such a case, the lineshape (13) acquires an extra factor  $1/\lambda_x\lambda_y$ , and the chemical potential  $\mu$  depends on  $\lambda_x\lambda_y$ , in addition to  $T$  and  $N$ .

We are interested primarily in the strongly degenerate Fermi gas ( $\beta E_F \gg 1$ ), for which the Sommerfeld expansion holds for the chemical potential:

$$\mu = E_F \left( 1 - \frac{\pi^2}{3} \left( \frac{1}{\beta E_F} \right)^2 \right) \quad (14)$$

In the opposite (non-degenerate gas) limit ( $\beta E_F \rightarrow 0$ ), one has  $\mu = \beta^{-1} \ln [(\beta E_F)^3 / 6]$ , and the lineshape (13) reduces to the Gaussian form if  $\Omega_g = \Omega_{ex}$ .

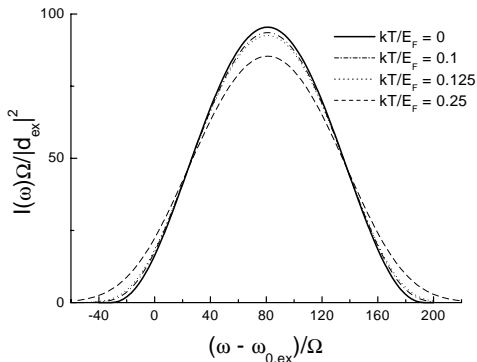


FIG. 4. Absorption lineshape at various temperatures for an isotropic trap with  $\Omega_g = \Omega_{ex}$ ,  $N = 10667$  and  $\alpha = 9$ .

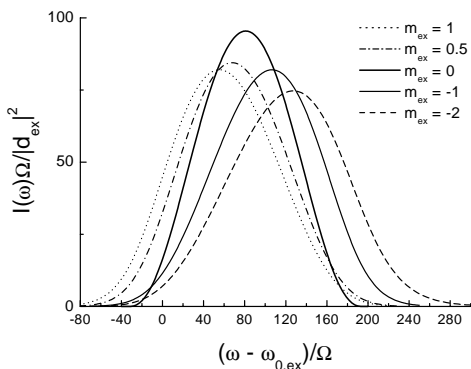


FIG. 5. Absorption lineshape for an isotropic trap with  $N = 10667$ ,  $\alpha = 9$ ,  $k_B T = 0.25 E_F$  and various  $\Omega_{ex}/\Omega_g$ .

Figure 4 shows the temperature dependence of the lineshapes for  $\Omega_g = \Omega_{ex}$ . At very low temperature ( $kT/E_F = 0.1$ ), the absorption spectrum is seen to be close to the zero-temperature limit. As the temperature increases, the absorption lineshape becomes broader and is no longer characterized by the  $(1 - z^2)^{5/2}$  behavior. Yet, the lineshape is still non-Gaussian, since the gas is strongly degenerate. Figure 5 shows the absorption spectrum for various values of  $m_{ex} = 1 - \Omega_{ex}^2/\Omega_g^2$ . For  $\Omega_{ex} > \Omega_g$ , the maximum position of the spectrum is shifted to larger frequencies. For  $\Omega_{ex} < \Omega_g$ , one has the opposite. Furthermore, one can see the obvious increase in the spectrum width if  $\Omega_{ex} > \Omega_g$ . This is due to the fact that an increase in the translational frequency of the elec-

tronically excited atoms leads to an increase and broader distribution of frequencies of the optical transitions.

In summary, we have studied the absorption spectrum by a cold gas of Fermi atoms using both the exact summation and also the Thomas - Fermi approximation. Oscillations have been obtained in the absorption spectrum calculated numerically for one-dimensional and anisotropic three-dimensional traps at a sufficiently small number of trapped atoms and  $T = 0$ . No such oscillations appear for the isotropic three-dimensional traps. Applying the TF approximation, relatively simple analytical expressions have been obtained for the lineshape of three dimensional traps at a sufficiently large number of trapped particles. At zero temperature, the approximated spectrum is characterized by a  $(1 - z^2)^{5/2}$  dependence. At non-zero temperature, the spectrum becomes broader, although remains non-Gaussian as long as the fermion gas is degenerate. The changes in the trap frequency for an electronically excited atom can introduce an additional line broadening.

The authors would like to express their appreciation for the useful discussions with H. Carmichael and T. Mossberg. This work has been completed during the Fulbright Scholarship by G. J. at the University of Oregon.

- 
- [1] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman and E.A. Cornell, *Science* **269**, 198 (1995).
  - [2] K.B. Davis, M.O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn and W. Ketterle, *Phys. Rev. Lett.* **75**, 3969 (1995).
  - [3] C.C. Bradley, C.A. Sackett, J.J. Tollett and R.G. Hulet, *Phys. Rev. Lett.* **75**, 3969 (1995).
  - [4] B. DeMarco and D.S. Jin, *Science* **285**, 1703 (1999).
  - [5] D.A. Butts and D.S. Rokhsar, *Phys. Rev. A* **55**, 4346 (1997).
  - [6] J. Schneider and H. Wallis, *Phys. Rev. A* **57**, 1253 (1998).
  - [7] B. DeMarco and D.S. Jin, *Phys. Rev. A* **58**, R4267 (1998).
  - [8] M. Lewenstein, L. You, J. Cooper and K. Burnett, *Phys. Rev. A* **50**, 2207 (1994).
  - [9] T. Wong, Ö. Müstecaplıoğlu, L. You and M. Lewenstein, *Phys. Rev. A* **62**, 033608 (2000).
  - [10] J. Javanainen and J. Ruostekoski, *Phys. Rev. A* **52**, 3033 (1995).
  - [11] T. Busch, J.R. Anglin, J.I. Cirac and P. Zoller, *Europhys. Lett.* **44**, 1 (1998).
  - [12] F. Gleisberg, W. Wonneberger, U. Schlöder and C. Zimmermann, e-print quant-physics/0009085 (2000).
  - [13] A. Orłowski, M. Gajda, P. Krekora, R.J. Glauber, J. Mostowski, *Opt. and Spectrosc.* **87**, 645 (1999).